Franklin Math Bowl 2008
Group Problem Solving Test – Grade 6

1. The fraction $\frac{32}{17}$ can be rewritten by division in the form $p + \frac{1}{1+\frac{q}{r}}$.

   Find the values of *p, q*, and *r.*

2. Robert has 48 inches of heavy gauge wire. He decided to cut the wire into pieces and solder them together to make a cubic frame as part of his science project. This cubic frame will be covered with Mylar to make a cube.
   a. If he uses all of the wire, what is the volume of the cube?
   b. How much Mylar is required to cover the frame?

3. A pegboard has pegs positioned one centimeter apart horizontally and vertically as pictured below. Sam ties a string from peg 2 to peg g and then to peg b and to peg p and to peg 4 and then back to peg 2. What is the length of the string **<u>and</u>** how much area is included inside the string?
   (If you want to draw the string in the picture to help, that would be o.k.)

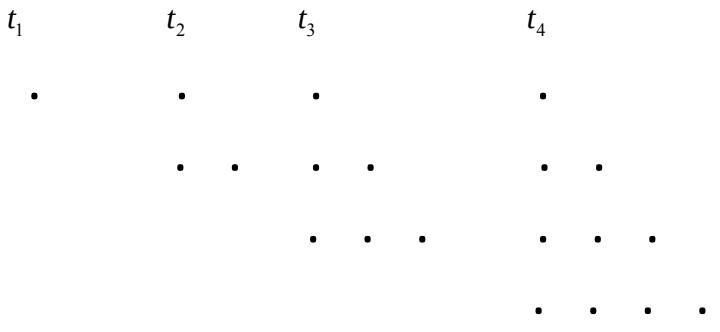
1.	**2.**	3.	**4**.	5.	6.
a.	**b.**	c.	d.	e.	f.
g.	h.	i.	j.	k.	l.
m.	n.	o.	**p.**	q.	r.

Franklin Math Bowl 2008
Group Problem Solving Test – Grade 7

1. A challenge was given to a designer to build a rectangular prism which had a volume of 462 cubic inches with all edge lengths being natural numbers and have the sum of the edge lengths be as small as possible. What is this sum? Explain how you know that this is the smallest sum.
   NOTE: Natural numbers are the numbers 1, 2, 3, 4, 5, . . .

2. At the market, grapefruit is displayed in a pyramid. There is 1 grapefruit at the top of the pyramid. There are 3 more grapefruit in the second level; 5 more grapefruit in the third level and 7 more grapefruit in the next level. This pattern continues to the bottom of the pyramid. The pyramid is 10 layers tall. How many grapefruit are in the pyramid?

3. Triangular numbers can be placed in a triangular array as pictured below. The first triangular numbers are 1, 3, 6, 10, . . . and designated as $t_1 = 1$, $t_2 = 3$, $t_3 = 6$ $t_4 = 10$, . . . These are created by adding 2, then 3, then 4, . . . Find the value of this sum, $t_{n-1} + t_n$, for any n.



Franklin Math Bowl 2008
Group Problem Solving Test – Grade 8

1. A series can be written as a sum of terms. For instance, the first 5 terms are written as $a_1 + a_2 + a_3 + a_4 + a_5$. Suppose the first term of a series is 1 and the sum of the first *n* terms of that series is $\frac{1}{n}$. Write a formula which can be used to find any term and use it to find term 20. That is, a_{20} = __?__.

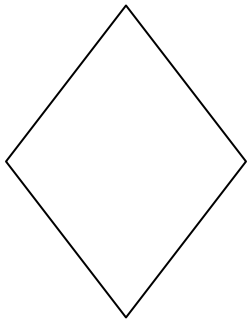
2. Write the first 4 prime numbers and use them as *a, b, c,* and *d* in the fractions $\frac{a}{b} + \frac{c}{d}$ to get the largest possible sum. Explain how you know that this is the largest possible sum.

3. A 20 pound pumpkin was originally 75% water. It sat in the sun for two days and was reduced to 70% water. How much did the pumpkin weigh at the end of the two days in the sun?

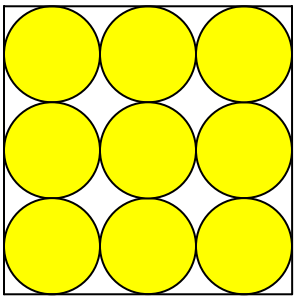
Franklin Math Bowl 2008
Group Problem Solving Test – Algebra

1. The graph of the line $ax + by + c = 0$ has a y-intercept at 6 and an x-intercept at p where $p > 0$. The line, the x-axis, and the y-axis form a triangle in the first quadrant which has an area of 12 square units. Find the integer values of a, b, and c which make this true and have no common factors.

2. A rhombus is a four sided figure with all sides of the same length as pictured at right. Diagonals of the rhombus connect opposite vertices and intersect at right angles at the midpoint of each diagonal. If the sides of the rhombus are all 10 inches and one diagonal is 4 inches longer than the other diagonal, what is the area of the rhombus?



3. Marta has designed a piece of jewelry composed of 9 gold disks, held together with platinum pieces between them. The diagram above shows her design. If platinum costs 5 times as much per square unit as the gold, what percent of the cost is used for the gold?



Franklin Math Bowl 2008
Group Problem Solving Test – Grade 6 Answers

1. By division $\frac{32}{17} = 1 + \frac{15}{17}$; $\frac{15}{17} = \frac{1}{\left(\frac{17}{15}\right)}$; $\frac{17}{15} = 1 + \frac{2}{15}$. So $\frac{32}{15} = 1 + \frac{1}{1 + \frac{2}{15}}$.

   Therefore, $p = 1,\ q = 2,$ and $r = 15.$

2. The cube has 12 edges, so each edge is 4 inches. This gives a cube with a volume of 64 cubic inches. There are 6 faces to the cube and each face has an area of 16 square inches. This gives a total surface of 96 square inches of Mylar used.

3. The lengths of the segments are determined by the Pythagorean Theorem. Their lengths are $\sqrt{5}$, $\sqrt{2}$, $\sqrt{8}$, 4, and 2. The length of the string is the sum of these numbers. This number is 9.65 centimeters. The simplest way to find the area inside the string is to think of the rectangle 3 cm wide and 3 cm tall and then subtract off the area of two triangles and a trapezoid. These areas are 1, 2, and 1.5. This leaves an area of 4.5 square cm.

Franklin Math Bowl 2008
Group Problem Solving Test – Grade 7 Answers

1. The prime factors of 462 are 2, 3, 7, and 11. Dimensions could be 1, 1, 462 which would give the largest sum or any 3 products that can be formed with these 4 numbers. The smallest sum occurs with 6, 7, and 11 for a sum of 24.

2. The number of grapefruit in each row is a perfect square. The numbers are 1, 4, 9, 16, 25, 36, …, 100 . These numbers added together gives 385 grapefruit.

3. A student might argue from numbers and count, but the simplest solution is to use the geometry and fit two consecutive sets of dots together to form a square. $t_1 + t_2 = 4$, $t_2 + t_3 = 9$. In general $t_{n-1} + t_n = n^2$. I would not expect a proof, but an argument which justifies their reasoning.

Franklin Math Bowl 2008
Group Problem Solving Test – Grade 8 Answers

1. Term one is 1. The sum of the first two terms is ½, so the second term is -1/2. The sum of the first three terms is ⅓, so the third term must add to ½ and give ⅓. Therefore term 3 is $\frac{-1}{6}$. This process continues and term *n* is $\frac{-1}{n(n-1)}$. This works for all terms after the first. Consequently term 20 is $\frac{-1}{380}$.

2. The only numbers that can be used are 2, 3, 5, and 7. The largest possible numbers are created by pairing the largest possible numerator with the smallest possible denominator. This means that the fractions should be $\frac{7}{2}+\frac{5}{3}$ for a sum of $\frac{31}{6}$.

3. The pumpkin originally weighed 20 pounds with 75% water. This means that 25 % is a solid which will not evaporate. That is, there are 5 pounds of solids. After two days, there are still 5 pounds of solids, but this is now 30% of the weight. With a proportion $\frac{5}{30}=\frac{x}{100}$. The pumpkin weighs $16\frac{2}{3}$ pounds.

Franklin Math Bowl 2008
Group Problem Solving Test – Algebra Answers

1. The height of the triangle is 6 and the base is p. Consequently, the area is $3p$ which is 12. Therefore, the value of p is 4. The equation $\frac{x}{4}+\frac{y}{6}=1$ is the equation of the line through the points (0,6) and (4,0). In the required form the equation is $3x + 2y = 12$. i.e. $a = 3$, $b = 2$, and $c = 12$.

2. The diagonals create 4 right triangles with legs of length x and $x + 2$ and a hypotenuse of 10. By the Pythagorean Theorem, $x^2 + (x + 2)^2 = 10^2$. $36 + 64 = 100$, so the legs of the triangles are 6 and 8. This gives 4 triangles each with area of 24 square inches. Consequently, the rhombus has an area of 96 square inches.

3. Only ratios are necessary, but to make the problem easier, suppose that the gold disks have a radius of 1 inch. This means that the total rectangle has dimensions of 6 inches by 6 inches or 36 square inches. Each disk has an area of π square inches; the total area of the disks is 9π square inches. This means that the platinum covers an area of 7.7256 square inches. The amount of gold is 4.6597 times the area of the platinum. This could have been done with only one circle in a square. Suppose that gold costs 1 unit and platinum costs 5 units of money. The one unit of platinum costs 5 and the 4.6597 units of gold costs 4.6597. $\frac{4.6597}{5+4.6597}$ gives 48.239% of the cost is for gold.