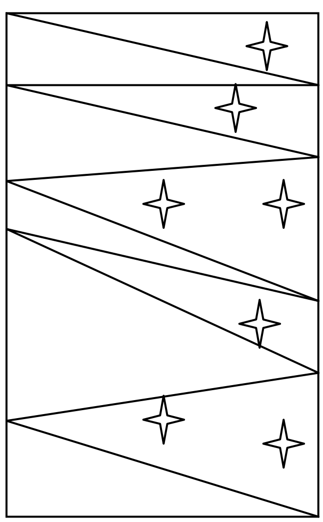
Franklin Math Bowl 2007
Group Problem Solving Test – 6th Grade

1. Consecutive integers are integers that increase by one. For example, 6, 7, and 8 are consecutive integers. If the sum of 9 consecutive integers is 9, what is the smallest integer in that set of integers?

2. A dartboard is made by drawing 5 concentric circles. The innermost circle has a radius of 1 inch; the second circle has a radius of 2 inches; the third circle has a radius of 3 inches; the fourth circle has a radius of 4 inches and the largest circle has a radius of 5 inches. The regions between the circles are colored. The smallest is colored red; the next is colored orange; the third region is colored yellow; the fourth region is colored green and the fifth region is colored blue. What is the ratio of the area of the green region to the combined areas of the yellow region and the orange region?

3.



Consider the rectangle pictured above and divided by line segments. What percent of the rectangle is in regions which contain stars? Explain how you arrived at your answer.

Franklin Math Bowl 2007
Group Problem Solving Test – 7th Grade

1. A Pythagorean triple is a set of numbers *a, b, c*, such that $a^2 + b^2 = c^2$ where c is the largest number. If $a, b,$ and c form a Pythagorean triple and $a^2 + b^2 + c^2 = 300$, what is the value of $a^2 + b^2$?

2. Find the sum of the numbers
1 – 2 + 3 – 4 + 5 – 6 + 7 – 8 + . . . -2006 + 2007. Explain how you found the sum without adding all of the terms.

3. The area of a square is 16 square feet. The square is converted into a circle by stretching its sides. By how much is the area of the square increased when it is converted into a circle?

Franklin Math Bowl 2007
Group Problem Solving Test – 8th Grade

1. A triangular array of numbers is given below.

				2				
			4		6			
		8		10		12		
	14		16		18		20	
22		24		26		28		30

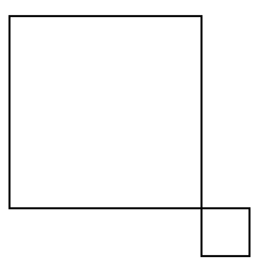
This pattern will continue row after row for 100 rows.

A. What is the first number in row 10? Explain how you can determine this number without writing the entire array of numbers.

B. What is the sum of the numbers in row 10? Show how you can determine this sum without writing the entire array of numbers.

2. The volume of a sphere is given by the formula $V = \frac{4\pi r^3}{3}$. If a sphere has radius *r*, what is the ratio of the volume of the sphere to the volume of the smallest cylinder which can contain it?

3. The figure below is composed of two squares which meet at a vertex and have parallel sides. The larger square has an area of 100 square centimeters and smaller square has an area of 4 square centimeters. What is the length of the longest line segment that can be drawn connecting two points in the figure? Justify your answer.

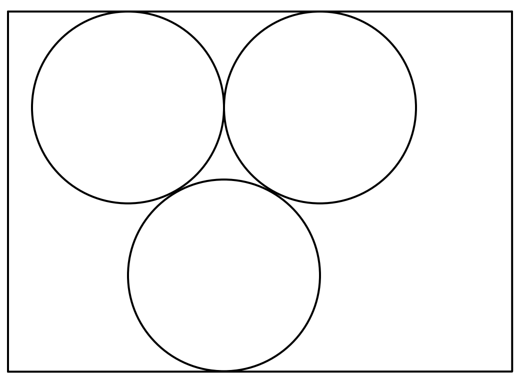


Franklin Math Bowl 2007
Group Problem Solving Test – Algebra I

1. If x is selected so that $x+\frac{1}{x}=6,$ what is the value of $x^3+\frac{1}{x^3}?$

2. A rectangle is positioned so that it has a vertex at the origin and the vertex opposite it is on the graph of $y=\frac{1}{x}$ for $x>0.$ If the height of the rectangle is twice the length of the base and the base is on the x-axis, what is the area of the rectangle?

3. If 3 hula hoops each have an outer diameter of 3 feet and they are placed so that they reach form one side of the table to the other as pictured below. How wide is the table? Note that the hula hoops are tangent to each other.



Franklin Math Bowl 2007
Answers to Problem Solving Test – 6th Grade

1. If the sum of the nine integers is nine, the average of the integers is 1. There are 4 integers above the average and 4 integers below the average. So the average integer is 1 and the four below it are 0, -1, -2, and -3. Similarly the four above are 2, 3, 4, and 5. The smallest integer is **-3**.

2. The area of a circle is πr^2, so the area of the green region is $16\pi - 9\pi$ or 7π square inches. The area of the yellow and orange regions is $9\pi - \pi$ or 8π square inches. So the ratio of the area of the green region to the combined areas of the yellow and orange regions is **7:8**.

3. There is **50%** of the rectangle in the marked sections. Each section is a triangle and the area of a triangle is half of the base times the height. The student's explanation must state that the bases of all of the triangles add up to the height of the rectangle. (The area of the starred triangles = the area of the plain triangles.)

Alternative solution to #1:

$$n + (n + 1) + \dots + (n + 8) = 9$$

So $9n + (1 + 2 + 3 + \dots + 8) = 9$

$$9n + \frac{8 \bullet 9}{2} = 9$$

$$9n + 36 = 9$$

$$n + 4 = 1$$

$$\underline{n = -3}$$

Franklin Math Bowl 2007
Answers to Problem Solving Test – 7th Grade

1. Since $a^2 + b^2 = c^2$, then $a^2 + b^2 + c^2 = 300$ is the same as $a^2 + b^2 + a^2 + b^2 = 300$ or $a^2 + b^2 = \boxed{150}$.

2. There are several methods which are logical. One can group the numbers as (1-2)+(3-4)+(5-6)+. . . +2007. This gives 1003 numbers of -1 plus the final 2007 for a sum of $\boxed{\textbf{1004}}$. One could also group the terms as 1 +(-2+3)+(-4+5)+. . . which gives 1004 terms of 1. It is also possible to look at 1+3+5+7 + . . .+2007 and from that subtract 2+4+6+8+. . . +2006 This gives allows the student to use Gauss's method of finding the average of each grouping and get the individual sums.

3. The side of the square will be 4 ft. Use the Pythagorean Theorem to find the diagonal of the square, which is also the diameter of the circle: $d^2 = 4^2 + 4^2 = 16 + 16 = 32$. So r^2 would be 32/4, or 8. That means the area of the circle, $A = \pi r^2 = 8\pi$. So the area of the square is increased by (area of circle – area of square), or $\boxed{8\pi - 16\text{ ft}^2}$.

Franklin Math Bowl 2007
Answers to Problem Solving Test – 8th Grade

1. Each entry is a multiple of 2 and they are listed in order. Row 1 has the number 2; row 2 begins with the number 4; row 3 begins with the number 8; row 4 begins with the number 14. These numbers increase by 2, then by 6, then by 8 and the pattern continues. The differences are linear. One can determine from the pattern that the first entry in row 10 is 92. The numbers at the end of the row are 2, 6, 12, 20,30, . . . These numbers increase by 4, then 6, then 8, then 10. The last entry in row 10 is 110. The numbers in row 10 increase by 2, so the sum will be the numbers 92 + 94 + . . . + 110 is **1010**.

(**A: <u>92</u>; B: <u>1010</u>**)

2. The cylinder would have height of $2r$ and radius of r. Its volume would be $\pi r^2 h$ or $\pi r^2(2r)$. The ratio of the volume of the sphere to the volume of the cylinder is $\dfrac{\frac{4}{3}\pi r^3}{2\pi r^3}$ so the volume of the cylinder is 2/3 the volume of the smallest cylinder which contains it.

3. The diagonals of the two squares are the longest segment in each square. The two diagonals which contain the common point will be in a line. [If they do not make this claim, it is not a complete argument.] The length of the larger square is 5 centimeters, so its diagonal is $5\sqrt{2}$ centimeters. The smaller square has side lengths of 2 centimeters and a diagonal of $2\sqrt{2}$ centimeters. The length of the line segment is $\boxed{7\sqrt{2}}$ or 9.899 centimeters.

Franklin Math Bowl 2007
Answers to Problem Solving Test – Algebra I

1. Consider $(x+\frac{1}{x})^3$. This is $x^3+\frac{1}{x^3}+3(x+\frac{1}{x})$. Since $x+\frac{1}{x}$ = 6, then $(x+\frac{1}{x})^3$ = 216. Therefore $x^3+\frac{1}{x^3}+3(x+\frac{1}{x})$ = $x^3+\frac{1}{x^3}$ + 18 = 216 and $x^3+\frac{1}{x^3}$ =216 – 18 = $\boxed{198}$.

2. The length of the base is *x*, so the altitude is 2*x*. That means that $2x=\frac{1}{x}$ or $x=\frac{1}{\sqrt{2}}$. So the area is $\boxed{\text{1 square unit}}$.

3. The radius of each hula hoop is 1.5 feet, so the centers of the hula hoops form an equilateral triangle with sides of length 3 feet. The altitude of this triangle is $1.5\sqrt{3}$. The width of the table is $\boxed{1.5+1.5\sqrt{3}+1.5}$ or $\boxed{\text{5.598 feet}}$.