FRANKLIN MATH BOWL

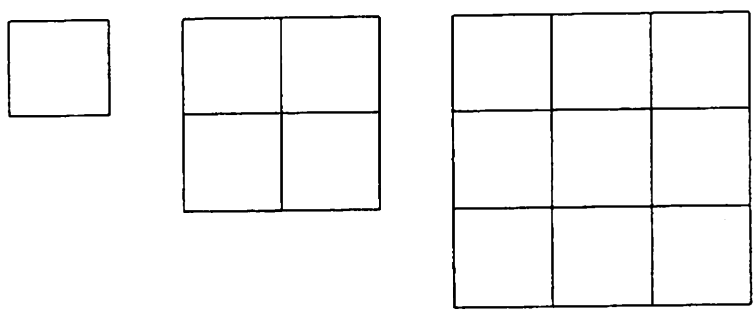
6th Grade Problem Solving Questions

2005

Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

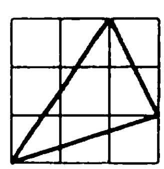
1. Find the smallest number greater than 2000 which is divisible by 2, 3, 4, 5, and 6. Explain how you arrived at this conclusion.

2. The square here is divided into smaller squares. Each square is of length 1. Below there are some examples of possible squares.



The 1 by 1 square contains one square. The 2 by 2 square contains 5 squares. One is a 2 by 2 square and 4 are 1 by 1 squares. How many squares of any size would be contained in a 3 by 3 square? How many squares of any size would be contained in a 10 by 10 square? Explain your reasoning. Give any examples that are needed in your explanation.

3. The triangle below is placed on a grid where every square is 1 centimeter by 1 centimeter. What percentage of the area in the 3 centimeter by 3 centimeter grid is inside the triangle? Explain how you arrived at your answer.



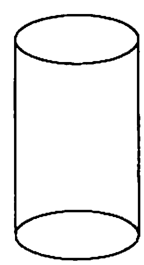
FRANKLIN MATH BOWL

7^{TH} Grade Problem Solving Questions

2005

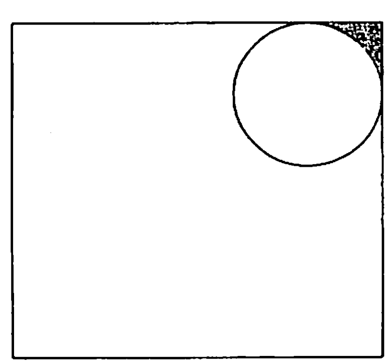
Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

1. A cylindrical can is designed to exactly hold three tennis balls with no extra space in the can. The sides of the can are made of cardboard and the ends of the can are made of metal. What is the ratio of the area of the metal part of the can to the cardboard part of the can? Below is a sketch of the can.



2. A cube has a volume of **P** cubic inches and a surface area of **P** square inches. (a) What is the length of an edge of the cube? (b) What is the length of a diagonal of the cube? A diagonal is the greatest possible distance between corners of the cube.

3. A circle with radius of 3 centimeters is placed in a square so that it is tangent to two sides of the square as pictured below. What is the area of the shaded part of the square in the upper right hand corner?



FRANKLIN MATH BOWL

8^{TH} Grade Problem Solving Questions

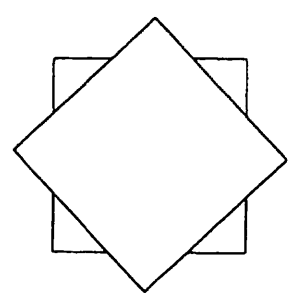
2005

Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

1. A function f is defined by $f(a,b) = a^{b+1} + 2b$. What is the difference between $f(4,3)$ and $f(3,4)$? Which is larger?

2. There are two numbers a and b with $a + b = ab = \frac{a}{b}$. Find these numbers.

3. The figure below is formed by drawing a square with sides of length 6 inches. A square of exactly the same size is rotated 45^o and placed on top of the square so that the center of the top square is on top of the center of the bottom square. What is the area of the visible part of the bottom square?



FRANKLIN MATH BOWL

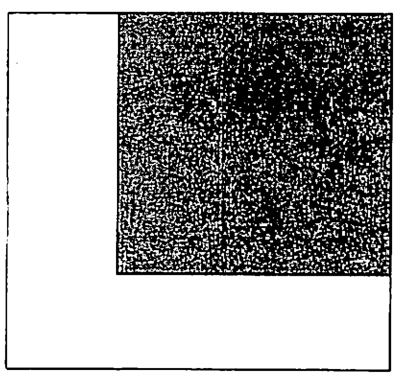
Algebra Problem Solving Questions

2005

Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

1. A quadratic equation ($ax^2 + bx + c = 0$) has roots which differ by one and $a + b + c = 12$. Write all possible quadratic equations. Show the work which leads to your answer.

2.



Sonya was given the white square above and she colored a smaller square in the corner of the original white square as is illustrated above. The area that she colored is the same as the area of the part which she left white. In terms of the length of the edge of the original square, how long is the edge of the colored square?

3. What restriction should you impose on b so that $x^2 - bx + 1 \geq 0$ for any x ?

Franklin Math Bowl

Answer Keys for Problem Solving Tests
2005

6th Grade Test

1. The least common multiple of all these numbers is 60. The smallest multiple of 60 that is larger than 2000 is (60)(34) = 2040.

2. The students should look at examples and conjecture that the number of squares is $1^2 + 2^2 + 3^2 + ... + n^2$ for an n by n square. Some explanation should be given for this counting system. A 3 by 3 square should have 14 squares. A 10 by 10 square would contain 385 squares.

3. The 3 by 3 grid that the triangle is placed on contains 9 squares, so its area is 9 square centimeters. There are 3 right triangles which are part of the grid and outside the triangle in question. The areas of these triangles are 1.5 square centimeters, 1 square centimeter and 3 square centimeters. The area of the marked triangle is 9 – 1.5 -1 – 3 or 3.5 square centimeters. The number 3.5 is **38.888%** of 9.

7th Grade Test

1. The radius of the tennis ball is r. The area of one end is πr^2, so the metal parts have a total area of $2\pi r^2$. The cardboard part has a circumference of $2\pi r$ and a height of $6r$. This gives an area for the cardboard part as $12\pi r^2$. Consequently the ratio of the areas of the metal part to the cardboard part is 1:6.

2. If the cube has an edge length of x inches, then its volume is x^3 and its surface area is $6x^2$. Since $x^3 = 6x^2$ we can conclude that x = 6 inches. By using the Pythagorean Theorem twice one can find that the diagonal is of length $6\sqrt{3}$ or 10.392 inches.

3. Consider a square with the given circle inscribed in it. The square will have sides of length 6 cm and an area of 36 cm^2. The circle has an area of 9π cm^2. This means that the four corners outside the circle have an area of $36 - 9\pi$ cm^2 and each corner has one-fourth of this area, i.e. $9 - \frac{9\pi}{4}$ which is approximately 1.931 cm^2. Another way of solving the problem is to draw perpendicular segments from the center of the circle to the original square.

8^{th} Grade Test

1. $f(4,3) = 4^{3+1} + 2 \cdot 3 = 262$, $f(3,4) = 3^{4+1} + 2 \cdot 4 = 251$; so the difference is 11 and $f(4,3)$ is larger.

2. Obviously $b \neq 0$. We can conclude that $a \neq 0$ too, because otherwise $a + b = ab$ leads to $b = 0$. On the other hand, $ab = a/b$ leads to $b = \pm 1$. The alternative $b = 1$ has to be discarded because we would have $a + 1 = a$, which is impossible. Thus we are left with $b = -1$, which in turn leads to $a - 1 = -a$, i.e. $a = 1/2$. In summary, $a = 1/2$ and $b = -1$ are the **only** numbers that satisfy the two given equalities.

3. The diagonal of the square is $6\sqrt{2}$ inches by the Pythagorean Theorem (8.485). The triangle that shows on the bottom square has altitude of one-half of 8.485-6 , i.e. 1.2426 inches. This is the altitude of an isosceles right triangle, so its area is 1.544 square inches. The four of them have an area of 6.1766 square inches.

Algebra Test

1. Let the roots be r and $r+1$. Then $(x-r)(x-r-1)=0$, which in turn leads to $x^2-(2r+1)x+r^2+r=0$. But $1-2r-1+r^2+r=12$, so $r^2-r-12=0$, i.e. $(r-4)(r+3)=0$. We note that $r=4$ leads to $(x-4)(x-5)=0$, i.e. $x^2-9x+20=0$. Analogously $r=-3$ leads to $(x+3)(x+2)=0$, i.e. $x^2+5x+6=0$. As expected, the problem could be solved by assuming that the roots are $r-1$ and r.

2. Denote by b the length of the original square. Let x be the length of the colored square. Then $x(b-x)+(b-x)b=x^2$. Thus $bx-x^2+b^2-bx=x^2$, i.e. $b^2=2x^2$. Therefore $x=b/\sqrt{2}$, which gives the approximate answer $x=0.707b$.

3. The parabola $y=x^2-bx+1$ has its vertex at $(\frac{b}{2}, 1-\frac{b^2}{4})$. Thus we need to have $1-\frac{b^2}{4} \geq 0$. Consequently $b^2 \leq 4$, i.e. $-2 \leq b \leq 2$.

Grade 6 — #3 2005

Area for the 3x3 grid = 9 → 2 pt

Area outside the Δ is found by finding the Area of 3 Δ's

$A_1 = \frac{1}{2}(2)(3) = 3$

$A_2 = \frac{1}{2}(1)(2) = 1$

$A_3 = \frac{1}{2}(3)(1) = 1.5$

5.5

→ 4 points

Area of Δ = 9 − 5.5 = 3.5 → 2 points

% Area is $\frac{3.5}{9} \times 100 = 33.888\%$ → 2 points

10 points