

FRANKLIN MATH BOWL
6th Grade Problem Solving 2003

Each team must have ONE complete write-up for each problem. Explain your reasoning. The problem will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

1. Sonia had a square piece of paper; its area was 16 square inches. Sonia folded the paper into 4 smaller squares and drew the largest possible circle in one of the smaller squares. What was the area of the circle?

2. A positive integer is called *highly composite* if it has more divisors than any positive integer that is smaller than it. For example, 4 is *highly composite* because it has 3 divisors, 1, 2, and 4. The smaller positive integers 1, 2, and 3 each have only 2 divisors.

List the first five *highly composite* positive integers and show that they are *highly composite*.

3. How many positive integers less than 200 have an odd number of factors? List them all and explain how you arrived at your answer.

FRANKLIN MATH BOWL
7th Grade Problem Solving 2003

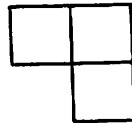
Each team must have ONE complete write-up for each problem. Explain your reasoning. The problem will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

1. Toothpicks are used to make the following designs.

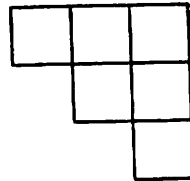
Design 1



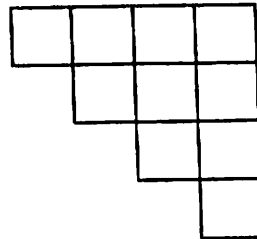
Design 2



Design 3



Design 4



How many toothpicks would be needed to make the 15th design? What is the area of that design?

2. The number 31752 can be factored as $2^3 \times 3^4 \times 7^2$. List all perfect squares which divide 31752 and explain how you know that you have found all of the perfect squares which divide 31752.

3. Augustus De Morgan said in 1864, "At some time in my life the square of my age was the same as the year in which I was that age." In what year was de Morgan born? Show how you arrived at your answer.

FRANKLIN MATH BOWL
8th Grade Problem Solving 2003

Each team must have ONE complete write-up for each problem. Explain your reasoning. The problem will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

1. A right triangle has two legs that form a right angle. The lengths of the legs are 3 inches and 12 inches. Use the hypotenuse of the right triangle as one side of a square. The region covered by the triangle and the square makes a pentagon. What is the area of the pentagon? Draw the figure and show how you determined your answer.

2. What is the units digit of 7^{2003} ? Write your answer with an explanation of how you made your decision.

3. A manufacturer produced a cube of cheese and coated it in red wax so that it would not dry out. When the cheese merchant received the cheese he cut it into 27 smaller cubes by making 2 evenly spaced cuts through each face of the original cube. He packaged these cubes individually.

- A. How many of the smaller cubes had no red faces?**
- B. How many of the smaller cubes had one red face?**
- C. How many of the smaller cubes had 2 red faces?**
- D. How many of the smaller cubes had 3 red faces?**
- E. How many of the smaller cubes had 4 red faces?**

Explain how you arrived at the answers.

FRANKLIN MATH BOWL
Algebra I Problem Solving 2003

Each team must have ONE complete write-up for each problem. Explain your reasoning. The problem will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.

1. In a mathematics class the girls had an average score of 83 on a test and the boys had an average score of 71. The class average was 80. What percent of the students were boys? Explain how you arrived at your answer.

2. Take a five digit number $abcde$. Write a six digit number $abcde1$ (The number 1 is put at the end of the original number.) and another six digit number $1abcde$ (The number 1 is placed at the beginning of the original number.). The six digit number with the one at the end is 3 times as large as the six digit number with the one at the beginning. What is the original five digit number? Explain how you arrived at your answer.

3. A box has 6 faces and 8 vertices. Three faces meet at each vertex. The faces that meet at one vertex have areas of 60 square inches, 90 square inches, and 150 square inches.

A. What is the volume of the box?

B. What are the lengths of each edge of the box?

FRANKLIN MATH BOWL

2003 Problem Solving Answer Keys

6th Grade

1. π square inches
2. They are 4 (3 divisors), 6 (4 divisors), 12 (6 divisors), 24 (8 divisors), and 48 (10 divisors)
3. There are 14 and they are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196

7th Grade

1. 270 toothpicks, the area is 120 square units
2. 4, 9, 36, 49, 81, 196, 324, 441, 1764, 3969, 15876
3. He was born in 1806. (In 1849 he was 43.)

8th Grade

1. The area is 171 square inches.
2. The units digit will be 3.
3. A. 1, B. 6, C. 12, D. 8, E. 0

Algebra

1. 25%
2. 42857
3. A. Volume is 900 cubic inches, B. 6 inches, 10 inches, and 15 inches